USN | | | | | | | | | | 10MCA12

First Semester MCA Degree Examination, December 2011
Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. For any three sets A, B and C, prove that
$A \cup (B \cap C) = (A \cup B) \cap (A \cup B)$ **(06 Marks)**

b. Prove that the set of rational numbers is countable. **(07 Marks)**

c. Let $f : A \to B$ and $g : B \to C$
i) If f and g are one to one, then prove that gof is one to one.
ii) If f and g are onto, then prove that gof is onto. **(07 Marks)**

2 a. Define: i) the principle of duality ii) tautology iii) contradiction. **(06 Marks)**

b. Prove that $\neg(r \vee (q \vee (\neg r \to \neg q))) \equiv \neg r \wedge (p \vee \neg q)$ using truth tables. **(07 Marks)**

c. Establish the validity of the following arguments:
$j \wedge (\neg j \wedge k)$
$j \to l$
$\therefore j \Leftrightarrow (l \wedge j)$ **(07 Marks)**

3 a. Define an open statement, universal and existential quantifiers, with an example for each. **(06 Marks)**

b. i) For $P(x) : |x| > 3$, $q(x) : x > 3$ and universe consists of all real numbers, write converse, inverse and contrapositive. **(03 Marks)**
ii) Show that $\forall x\, (p(x) \wedge q(x))$ and $\forall x\, p(x) \wedge \forall x\, q(x)$ are equivalent. **(04 Marks)**

c. If m is an even integer, then prove that m + 7 is odd. **(07 Marks)**

4 a. Prove by the mathematical induction that,
$2^n < n!$ for every positive integer n with $n \geq 4$. **(06 Marks)**

b. The Fibonacci numbers are defined recursively by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$, for $n \geq 2$.
Then prove that $F_0 + F_1 + \ldots\ldots\ldots + F_n = \sum_{i=0}^{n} F_i = F_{n+2} - 1$. **(07 Marks)**

c. Define one to one, onto and bijection function, with an example for each. **(07 Marks)**

5 a. Define the Stirling number of second kind. If m, n are positive integers with $1 < n \leq m$ then prove that $s(m+1, n) = s(m, n-1) + ns(m, n)$. **(06 Marks)**

b. State the pigeonhole principle. If $m \in z^+$ with m odd, then prove that there exists a +ve integer n, such that, m divides $2^n - 1$. **(07 Marks)**

c. Prove that a function $f : A \to B$ is invertible if and only if it is one to one and onto. **(07 Marks)**

6 a. Draw a Hasse diagram, representing a partially ordering { (a, b) | a divides b } on {1, 2, 3, 4, 6, 8, 12}. **(06 Marks)**

b. If R is an equivalence relation in a set A and x, y $\in$ A, then prove that i) x $\in$ [x] ii) x R y iff [x] = [y] iii) [x] = [y] or [x] $\cap$ [y] = ϕ. **(07 Marks)**

c. Define a group and a subgroup, with an example for each. **(07 Marks)**

7 a. State and prove the Lagrange's theorem on groups. **(06 Marks)**

b. Prove that every subgroup of a cyclic group is cyclic. **(07 Marks)**

c. The encoding function, $E: Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix, $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

i) Determine all code words. What can be said about the error-detection capabilities of this code? What about its error correction capacity?
ii) Find the associated parity-check matrix H.
iii) Use H to decode the following received word 00111. **(07 Marks)**

8 a. Define a ring, with an example. **(06 Marks)**

b. Let $E: Z_2^m \rightarrow Z_2^n$, m < n be the encoding function given by the generator matrix G or associated parity-check matrix H. Then prove that $c = E(Z_2^m)$ is a group code. **(07 Marks)**

c. Prove that a finite integral domain (D, +, ·) is a field. **(07 Marks)**

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